

**IM SYLLABUS (2008-2010)**

**PURE MATHEMATICS**

**IM 27**

***SYLLABUS***

<b>Pure Mathematics Syllabus</b>	<b>IM 27</b>	(Available in September) 1 Paper (3 hrs)
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**1. Aims:**

- To encourage confidence, enjoyment and satisfaction through the development and use of Mathematics.
- To develop an awareness of the relevance of Mathematics to other fields of study and of work.
- To be able to formulate a mathematical representation of a real life situation.
- To use appropriate technology such as computers and calculators as a mathematical tool.
- To develop abilities to reason logically and to construct mathematical proofs.

The syllabus assumes a good knowledge of the subject at SEC level and coverage of the extension topics in Paper 2A. It aims to consolidate this knowledge and to extend it to include more advanced concepts. Most of these new concepts are inclined to provide a general and coherent view of the subject but they will not be treated with the same rigour and at the same depth as in the Advanced level course in the subject.

**2. Assessment objectives:**

- To demonstrate the understanding of mathematical concepts, theories and techniques
- To recognise and apply standard models in different contexts.
- To use calculator technology and formulae booklets efficiently.

**3. Scheme of Assessment:**

There will be one examination paper of three hours. The paper will contain questions of possibly varying difficulty. Marks allotted to each question will be shown.

No question should carry more than 15 marks. Ten questions will be set and candidates are required to answer all questions. The total number of marks will be 100. Any examination question can contain material on more than one topic.

Scientific calculators may be used but all necessary working must be shown. The use of Graphical calculators is however prohibited. Disciplinary action will be taken against students making use of such calculators. A booklet with mathematical formulae will be provided.

**4. Grade Description:**

## Grade A

- Candidates who are able to recall and select almost all concepts and techniques required in various contexts.
- Candidates who derive results to a high degree of accuracy.
- Candidates who can use mathematical language correctly and able to proceed logically in the solution of almost all of the set problems.

## Grade C

- Candidates who are able to recall and select most of the concepts and techniques required in various contexts.
- Candidates who derive results to an appropriate degree of accuracy.
- Candidates who can use mathematical language correctly and able to proceed logically in the solution of most of the set problems.

## Grade E

- Candidates who are able to recall and select some of the concepts and techniques required in various contexts.
- Candidates who derive results to an acceptable degree of accuracy.
- Candidates who can use mathematical language correctly and able to proceed logically in the solution of a fair number of the set problems

**5. Subject content:**

The topics in the syllabus are not arranged in teaching order. The syllabus is not meant as a teaching scheme and teachers are free to adopt a teaching sequence that they deem to be suitable for their students.

Topics	Notes
1. Positive and negative rational indices. Laws of indices. Laws of logarithms	Solution of simple equations involving indices and logarithms of the form $a^{px+q} = b$ only. Problems involving change of base will not be set.
Use and manipulation of surds.	
2. Polynomials, rational functions, factor and remainder theorems. Simple partial fractions.	Simplification of rational expressions including factorising and cancelling, and algebraic division.  Problems on partial fractions could include denominators such as: $(ax+b)(cx+d)$ , $(ax+b)(cx+d)^2$ and $(ax+b)(cx^2+dx+e)$ . The degree of the denominator must not be greater than <i>three</i> .
3. The quadratic equation in one variable.	Solution of the quadratic equation by completing the square or the use of the formula. Nature of roots. Knowledge of the relation between the roots $\alpha$ , $\beta$ and the coefficients of a quadratic equation. Forming new equations with roots related to the original. Calculation of expressions such as: $\alpha^2 + \beta^2$ , $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ . Problems involving the expression $\alpha^3 + \beta^3$ will not be set.
4. Arithmetic series, finite and infinite geometric series. Pascal's triangle. The binomial expansion for positive integral indices.	The general term and the summation of an arithmetic and geometric series. Arithmetic and geometric mean. Knowledge of the notation $\sum$ and $\binom{n}{r}$ .

Topics	Notes
<p>5. Simple counting problems involving permutations and combinations. Applications to simple problems in probability</p>	<p>The knowledge of probability expected will be limited to the calculation of probabilities arising from simple problems of enumeration of equally likely possibilities, including simple problems involving the probability of the complement of an event and of the union and intersection of two events.</p> <p>Knowledge of the rules:  <math>P(A') = 1 - P(A)</math>,  <math>P(A \cup B) = P(A) + P(B) - P(A \cap B)</math></p> <p>is expected</p> <p>Questions on conditional probability will not be set.</p> <p>Tree and Venn diagrams may be used and candidates are expected to understand set notation.</p>
<p>6. Plane Cartesian coordinates. Distance between two points and the perpendicular distance from a point to a line. Elementary treatment of lines. Intersection of a straight line with a curve.</p> <p>Reduction of a relation to linear form, and graphical determination of the constants</p>	<p>Relations will be limited only to equations of the form:</p> $y = ab^x, \quad y = ax^b$ <p>and</p> $\frac{1}{y} = \frac{1}{x} + \frac{1}{a}.$
<p>7. The concept of a function as a mapping: domain and range.</p> <p>The exponential and logarithmic functions and their graphs.</p> <p>The six trigonometric functions</p> <p>Solution of simple trigonometric equations.</p> <p>The identity <math>\cos^2 \theta + \sin^2 \theta \equiv 1</math> and simple corollaries.</p>	<p>Use of function notations, e.g. <math>f(x) = 3x - 5</math>, <math>f : x \rightarrow 3x - 5</math>.</p> <p>Graphs of functions of the type <math>\sin nx</math>, <math>a \cos x</math>, <math>\tan(x + \alpha)</math>, where <math>n</math> is a positive integer. Graphs of reciprocal trigonometric functions are not required.</p> <p>Trigonometric equations of the type <math>a \sin(x + \alpha) = c</math> and <math>a \cos nx = c</math>, where <math>n</math> is a positive integer only. Quadratic trigonometric equations may be included. General solutions are not required.</p> <p>Other trigonometric identities and the addition theorems are not required.</p>
<p>8. Radian measure. Knowledge of the values of cosine, sine,</p>	

Topics	Notes
and tangent of $\pi/k$ , $k = 1, 2, 3, 4, 6$ in surd or rational form. Use of the formulae: $s = r\theta$ , $A = \frac{1}{2}r^2\theta$ .	
9. Simple curve sketching.	Curve sketching of polynomials not higher than the third degree.
Transformations.	Knowledge of the effect of the simple transformations $y = f(x+a)$ , $y = f(x) + a$ , $y = f(ax)$ , $y = af(x)$ for $a > 0$ , $y = f(-x)$ , $y = -f(x)$ . Transformations of exponential, logarithmic, trigonometric and polynomial functions. Combinations of transformations will not be required.
Simple inequalities in one variable.	Graphical or algebraic solution of inequalities such as the following : (i) $(x-a)(x-b)(x-c) > 0$ , (ii) $(x-a)/(x-b) < c$ .
10. The derivative as a limit. Differentiation of sums, products, quotients and composition of functions. Differentiation of algebraic, trigonometric, exponential and logarithmic functions.  Applications of differentiation to gradients, tangents, maxima and minima, points of inflexion and curve sketching.  Simple problems on rates of change.	A rigorous treatment of limits is not expected.  Inverse trigonometric functions are excluded.  Differentiation of implicit and parametric functions is not required. Second order derivatives are not required.  Problems involving the chain rule of the type $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ may be set.
11. Integration as the limit of a sum and as the inverse of differentiation.	A rigorous treatment is not required.

Topics

The evaluation of integrals by means of standard forms and by partial fractions.

Notes

Integrals of the functions  $x^n$ ,  $\frac{1}{x}$ ,  $\frac{1}{ax+b}$ ,  $e^{kx}$ ,  $\sin kx$  and  $\cos kx$ .

Use of the result  $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$ .

Evaluation of integrals by substitution and by parts is not required.

Definite integrals. Application of integration to the calculation of areas.

To include finding the area of a region between two curves and between a line and a curve.

12. First order differential equations with separable variables.

Problems requiring the formation of a differential equation will not be set.

13. The algebra of matrices.  
Addition and multiplication.  
Distributivity of multiplication over addition. Associativity. The zero matrix and the identity matrix. Non-commutativity of multiplication. The inverse of a matrix.

Students will only be expected to be able to find inverses of  $2 \times 2$  matrices but they should be able to verify that, say, two given  $3 \times 3$  matrices are inverses of each other.

Applications will be limited to linear transformations in the plane.  
Students are expected to find the matrices of the following transformations:

- a) enlargement,
- b) rotation through multiples of  $90^\circ$ ,
- c) reflections in the lines  
 $x = 0$ ,  $y = 0$ ,  $y = x$  and  $y = -x$ .

## Notation

The following notations are to be used in the Intermediate and Advanced Level Pure Mathematics examinations.

### 1. Set Notation

$\in$	is an element of
$\{x_1, x_2, \dots\}$	the set with elements $x_1, x_2, \dots$
$\{x : \dots\}$	the set of all $x$ such that $\dots$
$A'$	the complement of the set $A$
$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \dots\}$
$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \dots\}$
$\mathbb{Q}$	the set of rational numbers, $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{N}\right\}$
$\mathbb{R}$	the set of real numbers
$\cup$	union
$\cap$	intersection
$[a, b]$	the closed interval, $\{x \in \mathbb{R} : a \leq x \leq b\}$
$[a, b), [a, b[$	the interval, $\{x \in \mathbb{R} : a \leq x < b\}$
$(a, b], ]a, b]$	the interval, $\{x \in \mathbb{R} : a < x \leq b\}$
$(a, b), ]a, b[$	the open interval, $\{x \in \mathbb{R} : a < x < b\}$

### 2. Miscellaneous Symbols

$=$	is equal to
$\neq$	is not equal to
$\equiv$	is identical to or is congruent to
$\approx$	is approximately equal to

$\propto$	is proportional to
$<$	is less than
$\leq$	is less than or equal to
$>$	is greater than
$\geq$	is greater than or equal to
$\infty$	infinity

### 3. Operations

$a + b$	$a$ plus $b$
$a - b$	$a$ minus $b$
$a \times b, ab, a.b$	$a$ multiplied by $b$
$a \div b, \frac{a}{b}, a/b$	$a$ divided by $b$
$\sum_{i=1}^n a_i$	$a_1 + a_2 + \dots + a_n$
$\sum_{i=k}^n a_i$	$a_k + a_{k+1} + \dots + a_n, \quad k \leq n$
$\sqrt{a}$	the positive square root of $a$
$ a $	the modulus of $a$
$n!$	$n$ factorial
$\binom{n}{r}$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n \in \mathbb{N}$ $\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbb{Q}$

### 4. Functions

$f(x)$	the value of the function $f$ at $x$
$f : A \rightarrow B$	$f$ is a function under which each element of set $A$ has an image in set $B$
$f^{-1}$	the inverse function of the function $f$
$g \circ f, gf$	the composite function of $f$ and $g$ which is defined by $(g \circ f)(x)$ or $gf(x) = g(f(x))$

$\lim_{x \rightarrow a} f(x)$	the limit of $f(x)$ as $x$ tends to $a$
$\Delta x, \delta x$	an increment of $x$
$\frac{dy}{dx}$	the derivative of $y$ with respect to $x$
$\frac{d^n y}{dx^n}$	the $n$ th derivative of $y$ with respect to $x$
$f'(x), f''(x), \dots, f^{(n)}(x)$	the first, second, ... , $n$ th derivative of $f(x)$ with respect to $x$
$\int y \, dx$	the indefinite integral of $y$ with respect to $x$
$\int_a^b y \, dx$	the definite integral of $y$ with respect to $x$ between the limits $x = a$ and $x = b$
$\dot{x}, \ddot{x}, \dots$	the first, second, ... derivatives of $x$ with respect to $t$

## 5. Exponential and Logarithmic Functions

$e$	base of natural logarithms
$e^x, \exp x$	exponential function of $x$
$\log_a x$	logarithm to the base $a$ of $x$
$\ln x, \log_e x$	natural logarithm of $x$
$\lg x, \log_{10} x$	logarithm of $x$ to base 10

## 6. Circular and Hyperbolic Functions

$\sin, \cos, \tan$	} the circular functions
$\operatorname{cosec}, \sec, \cot$	
$\arcsin, \arccos, \arctan$	} the inverse circular functions
$\operatorname{arccosec}, \operatorname{arcsec}, \operatorname{arccot}$	

$\left. \begin{array}{l} \sinh, \cosh, \tanh \\ \operatorname{cosech}, \operatorname{sech}, \operatorname{coth} \end{array} \right\}$	the hyperbolic functions
$\left. \begin{array}{l} \operatorname{arsinh}, \operatorname{arcosh}, \operatorname{artanh} \\ \operatorname{arcosech}, \operatorname{arsech}, \operatorname{arcoth} \end{array} \right\}$	the inverse hyperbolic functions

## 7. Complex Numbers

$i$	square root of $-1$
$z$	a complex number, $z = x + iy$
$\operatorname{Re} z$	the real part of $z$ , $\operatorname{Re} z = x$
$\operatorname{Im} z$	the imaginary part of $z$ , $\operatorname{Im} z = y$
$ z $	the modulus of $z$ , $ z  = \sqrt{(x^2 + y^2)}$
$\arg z$	the argument of $z$ , $\arg z = \theta$ , $-\pi < \theta \leq \pi$
$z^*$ or $\bar{z}$	the complex conjugate of $z$ , $z^* = x - iy$

## 8. Matrices

$\mathbf{M}$	a matrix $\mathbf{M}$
$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
$\mathbf{M}^T$	the transpose of the matrix $\mathbf{M}$
$\det \mathbf{M}$ or $ \mathbf{M} $	the determinant of the square matrix $\mathbf{M}$

## 9. Vectors

$\mathbf{a}$	the vector $\mathbf{a}$
$\overrightarrow{AB}$	the vector represented in magnitude and direction by the directed line segment $AB$
$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the direction of the Cartesian coordinate axes

$ \mathbf{a} , a$	the magnitude of $\mathbf{a}$
$ \overrightarrow{AB} , AB$	the magnitude of $\overrightarrow{AB}$
$\mathbf{a} \cdot \mathbf{b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
$\mathbf{a} \times \mathbf{b}$ or $\mathbf{a} \wedge \mathbf{b}$	the vector product of $\mathbf{a}$ and $\mathbf{b}$

## 10. Probability

$A, B, C, \text{ etc}$	events
$A \cup B$	union of the events $A$ and $B$
$A \cap B$	intersection of the events $A$ and $B$
$P(A)$	probability of the event $A$
$A'$ or $\bar{A}$	complement of the event $A$
$P(A B)$	probability of the event $A$ conditional on the event $B$